LOGICAL EQUIVALENCES

Monday, September 24 2012

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September 23, 2012

Tautology and Contradiction

- Tautology is a compound proposition that is always true, no matter what the truth values of it's underlying propositions.
- Contradiction is a compound proposition that is always false, no matter what the truth values of it's underlying propositions.
- Contigency is a proposition that is neither tautology and contradiction.

Example

Identify which of following sentences are tautology, contradiction, and contigency

1. $[x > 2] \wedge [x = 2]$
2. I have to go now and my class is taking place now
3. $p \vee \neg p$ where p is any proposition
4. $p \wedge \neg p$ where p is any proposition
5. $p \rightarrow q$, $p \oplus q$

- Two compound propositions that have the same truth values in all possible cases are called **logically equivalent**. On other words, the proposition p and q are called logically equivalent if biimplicaton $p \leftrightarrow q$ is a tautology. Furthermore, the notation $p \equiv q$ or $p \Leftrightarrow q$ denotes that p and q are logically equivalent.

Example

Let P be the implication $p \rightarrow q$ and Q be the contraposition $\neg q \rightarrow \neg p$. Prove that $P \equiv Q$.

Proof.

Create a truth table for $p \rightarrow q$ and $\neg q \rightarrow \neg p$, and check the truth values in all cases. □

Fact

The logically equivalent is intended for two compound propositions, but the tautoloy is addressed for one compound proposition.

Some Basic Form of Logical Equivalences

1. Identity laws : $p \wedge T \equiv p$ dan $p \vee F \equiv p$ where T and F are TRUE and FALSE proposition, respectively .
2. Domination laws : $p \vee T \equiv T$ dan $p \wedge F \equiv F$.
3. Idempotent laws : $p \vee p \equiv p$ dan $p \wedge p \equiv p$.
4. Doble negation laws : $\neg(\neg p) \equiv p$.
5. Comutative laws : $p \vee q \equiv q \vee p$ dan $p \wedge q \equiv q \wedge p$.
6. Assosiative laws : $(p \vee q) \vee r \equiv p \vee (q \vee r)$ dan $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$.
7. Distributive laws : $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ dan $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.
8. De Morgan's laws : $\neg(p \vee q) \equiv \neg p \wedge \neg q$ dan $\neg(p \wedge q) = \neg p \vee \neg q$.
9. Absorption laws : $p \vee (p \wedge q) \equiv p$ dan $p \wedge (p \vee q) \equiv p$.
10. Negation laws : $p \vee \neg p \equiv T$ dan $p \wedge \neg p \equiv F$.

These basic logical equivalences can be proven by mean of the truth table.

Example

Prove that $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.

Proving without Truth Table

Example

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent without using the truth table.

Proof.

Justify which the basic rules are used in the steps

$$\begin{aligned}
\neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge (\neg(\neg p \wedge q)) \,[\text{step } 1] \\
&\equiv \neg p \wedge (\neg(\neg p) \vee \neg q) \,[\text{step } 2] \\
&\equiv \neg p \wedge (p \vee \neg q) \,[\text{step } 3] \\
&\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) \,[\text{step } 4] \\
&\equiv F \vee (\neg p \wedge \neg q) \,[\text{step } 5] \\
&\equiv (\neg p \wedge \neg q) \,[\text{step } 6]
\end{aligned}$$

□

Example

Prove that $(p \wedge q) \to (p \vee q)$ is a tautology by an algebraic derivation.